Université Lille Nord de France
Pôle de Recherche et d'Enseignement Supérieur

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Titre : Intégration projective classique et macroscopique pour des équations cinétiques dans leurs limites fluides et diffusives.

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Descriptif :
Kinetic theory is a cornerstone of modern physics. **Kinetic equations** usually represent a **gas** as a set of particles undergoing instantaneous collisions interspersed with ballistic motion. Nowadays, these models appear in a variety of sciences and applications, such as **astrophysics**, **aerospace** and nuclear engineering, **semiconductors**, fusion processes in **plasmas**, as well as **biology**, finance and social sciences. The common structure of such equations consists in a combination of a linear **transport** term with one or more stiff **interaction** terms, which together dictate the time evolution of the distribution of particles in the (six-dimensional) position-velocity phase space.
Due to the **high dimensionality** of the phase-space, and the **nonlinear** phenomena dictating the interactions, it is desirable to develop very **efficient** numerical methods for solving this type of equation. The **Asymptotic Preserving** (AP) strategy consists in designing numerical schemes valid both in kinetic and limit regimes (such as **fluid** of **diffusive**), independently of the discretization parameter. Moreover, the obtained discretizations should avoid inversion of complicated nonlinear and nonlocal operator and be almost explicitly solvable. Various discretizations of the classical kinetic equations (such as the **Boltzmann** or **BGK** models) exist, but the most important differences appear in the choice of the time discretization.
A general time discretization technique called **Projective Integration** (PInt) was introduced to overcome some of these difficulties. It is designed to numerically solve systems of **nonlinear and stiff ODEs**, obtained e.g. as a semi-discretized form of a partial differential equation. PInt aims at mimicking the continuous behavior of this ODE system, namely that because of some stiffness, the solution will be projected on a **slow**, low dimensional **manifold** in a very short time. The formal idea is then to take a small number of time steps of an inner integrator, with a time scale corresponding to the fast rate of **damping** of the components of the solution on the fast manifold. Then, a forward **extrapolation** is performed with a large time step, corresponding to the remaining components of the solution living on the slow manifold. The inner integrator can be **explicit** because its time steps will be chosen very small (comparable with the stiffness). Such a strategy has been used with great success for linear and nonlinear kinetic equations in both the diffusive and hyperbolic scalings in a series of papers of the thesis project adviser.

Objective.

1. To tackle the more general case of the **multiple species Boltzmann equation**. Indeed, it is believed that realistic numerical simulations of e.g. the space shuttle re-entry would require to couple up to 20 different kinetic equations in order to take into account the atmospheric chemistry ($O_2 - CO_2 - H_2 -$

CH4 recombinations mostly). Mathematically, this makes the development of PInt method more difficult, because of the very intricated structure of the collision operator. Nevertheless, some very recent works precised this structure, and a numerical method for efficiently computing the collision operators was developed. Using all these results, the development of a PInt method for this type of model is planned, paving the way to massively parallel realistic simulations of space shuttles re-entry.

2. To develop PInt method for the biological ``Run-and-Tumble'' chemotactic motion model. It accurately describes bacterial chemotaxis using a kinetic equation with a linear collision operator, coupled with a nonlinear Poisson equation. A numerical method AP in the diffusive limit for this model has been recently developed, but is only first order in time. Adapting the PInt method to this problem to obtain an arbitrarily high order method for this problem would then be very profitable for applications. The mathematical study of the structure of this chemotaxis model is envisaged, along with modifications of the Pint method because of the possibility of blowup in finite time for this model.